Prime numbers and base 10^*m* positional numeral systems.

1. Abstract.

This paper holds a study into verifying large prime numbers with a right-to-left base 10^*m* positional numeral system, without using the operators multiplying and division.
Large integers, like recent Mersenne primes, are broken into pieces based on a base 10^*m* numeral system.
The principles of the Sieve of Eratosthenes are used to close in on the integer to be checked, often making huge leaps.
Describing the proces is simplified by using 32-bit integers and base 10^2 or 10^8 numeral systems, while modern computers nowadays use 64-bit integers and can accommodate a 10^{18} numeral system.

The base 10^*m* numeral system evolved from the (double) primorial sieve, which is one of the results of the hunch to place a prime number on the short leg of a primitive Pythagorean triangle. Other finds in that project where the "The Ulam spiral unraveled", "the last digit gap between prime numbers", the "segmented prime spirals", etc.

2. Principles of a positional numeral base systems.

A positional numeral system is a system to write numbers in an other base then the usual base 10 numeral system.
There are several positional numeral base systems, see for examples figure 2a below.

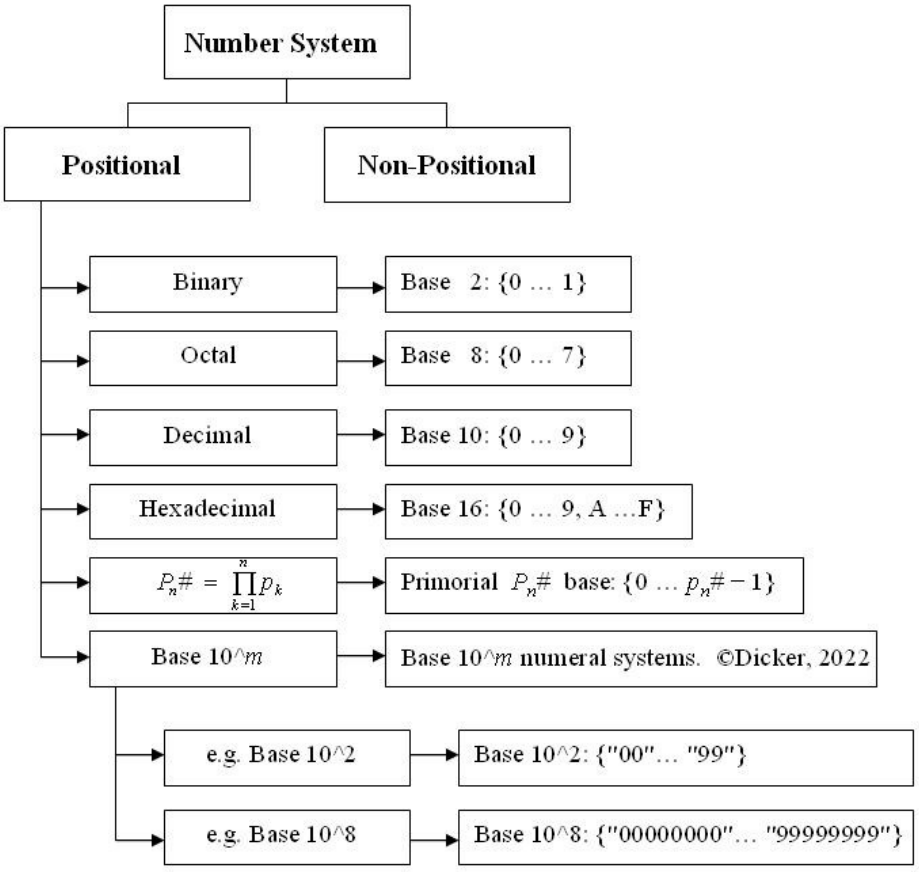

Fig. 2a: Overview of some positional numeral base systems, including the base 10^*m*.

In all base numeral systems: $number_b = [a_N : ... : a_2 : a_1 : a_0]_b = (a_N b^N + ... + a_2 b^2 + a_1 b^1 + a_0 b^0)_{10} = g_{10}$

With:
- b = base of then numeral system
- a_n = the "value" of the n-th position (right-to-left), with $0 \le a < b$ and $n \ge 0$
- n = starts with 0 for natural numbers
- $N + 1$ = the total number of positions / the number of value places
- g_{10} = number written in base 10 = $(a_N b^N + ... + a_2 b^2 + a_1 b^1 + a_0 b^0)_{10}$

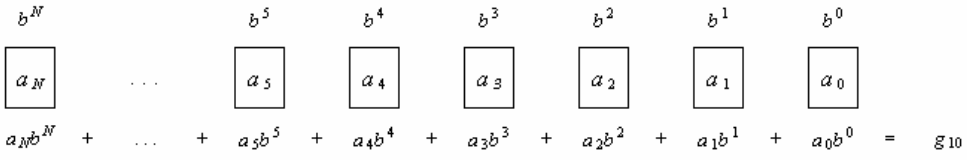

Fig. 2b: Mathematical principle of a right-to-left base numeral system.

The Octal numeral system is a well known right-to-left positional numeral base system.
In the base 8 positional numeral system each position has the value a_n with $0 \le a_n < b$ and $b = 8$.

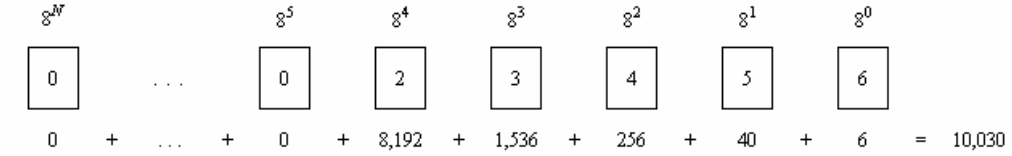

Fig. 2c: The Octal numeral system and the conversion of a Octal number to a decimal number.

In fig. 2c the number $[2 : 3 : 4 : 5 : 6]_8 = 2 \bullet 8^4 + 3 \bullet 8^3 + 4 \bullet 8^2 + 5 \bullet 8^1 + 6 \bullet 8^0 = 10{,}030_{10}$
So, the number $g = 10{,}030_{10} = 23456_8 = G[2 : 3 : 4 : 5 : 6]_8$

The primorial $p_3\#$ is the product of the first three prime numbers, e.g. $p_3\# = p_1 \bullet p_2 \bullet p_3 = 30$
A base $p_3\#$ numeral system could be used as a right-to-left positional numeral base system.

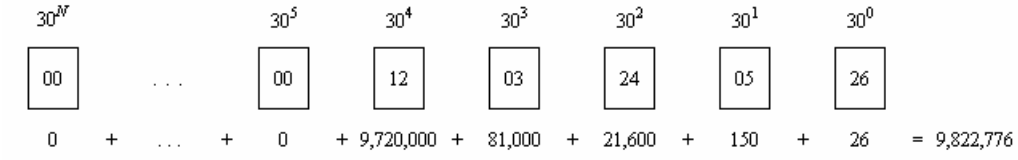

Fig. 2d: Full implementation of a right-to-left base $p_3\#$ numeral system.

In a full $p_3\#$ right-to-left positional numeral system each position has the value a_n with $0 \le a_n < b$
and $b = p_3\#$, the base 30.
For example:
the number [12:03:24:05:26] (base $p_3\#$) = $12 \bullet 30^4 + 3 \bullet 30^3 + 24 \bullet 30^2 + 5 \bullet 30^1 + 26 \bullet 30^0 = 9{,}822{,}776_{10}$

3. The base 10^*m* positional numeral system.

3.1. Practical use of the base 10^*m* positional numeral system.

Almost all modern CPU/GPUs have a division instruction. As it works on the default word size, it doesn't matter how fast it is, in terms of Big-O it is constant, so its always O(1). The exception are operations performed on non-word sized data. This happens e.g. when talking about BigInt libararies. In this case ALL operations (addition, multiplication,...) are no longer O(1) but depend on the size of the numbers.

In 2017 the largest Mersenne prime is $M_{77232917} = 2^{77232917} - 1$ with 23,249,425 digits when written in base 10.
The new base 10^*m* positional numeral system is developed to perform mathematical operations on very large integers.

In the base 10^8 positional numeral system, with $b = 10^8$, each position n has an integer value a_n with $0 \leq a_n < b$.
The base 10^8 notation is represented by an eight digit decimal number per position.
A long word 4 byte Integer array has $2^{31} - 1 \approx (2^{10})^3 \approx (10^3)^3$ positions to store each a_n (base 10^8).
The array thus can hold integers up to $(10^8)^{(2^{31} - 1)}$, so some 2,000 million digits when written in base 10.
Note: the square root of an integer in a base 10^*m* positional numeral system with $m \equiv 1 \pmod 2$ does not work well.

The 10th Mersenne prime is $g = M_{89} = 2^{89} - 1 = 11111111 \ldots 11111111_2$.
$M_{89} = 618{,}970{,}019{,}642{,}690{,}137{,}449{,}562{,}111_{10} = 0.618... \bullet 10^{27}$ and has 27 digits when written in base 10.
In the base 10^8 positional numeral system, $M_{89} = g = G[\ 00000618 : 97001964 : 26901374 : 49562111\ ]$ (base 10^8).
So $g = M_{89} = 618 \bullet (10^8)^3 + 97001964 \bullet (10^8)^2 + 26901374 \bullet (10^8)^1 + 49562111 \bullet (10^8)^0$.

$(10^8)^N$		$(10^8)^3$		$(10^8)^2$		$(10^8)^1$		$(10^8)^0$
	+ ... +	G(003) 00000618	+	G(002) 97001964	+	G(001) 26901374	+	G(000) 49562111

Fig. 3.1a: The 10th Mersenne prime written in the base 10^8 positional numeral system.

By writing integers in the base 10^8 positional numeral system, they are no longer treated as a solid integer.
The Sieve of Eratosthenes is now ideally suited to verify if a very large integer g is divisible by a divisor $d < \sqrt{g}$.
In the base 10^8 positional numeral system, the normal steps of going through all odd multiples of a divisor, can be significantly reduced to a small fraction of the regular steps needed.

In this section the 25th Mersenne prime M_{21701} is chosen to demonstrate the use of the base 10^8 numeral system.
The 25th Mersenne prime $M_{21701} = 2^p - 1 = 2^{21701} - 1 \approx 0.44867 \bullet 10^{6533} = 44{,}867 \bullet 10^{6528} = 44{,}867 \bullet (10^8)^{816}$.
The natural number M_{21701} has 6,533 digits when written in base 10. M_{21701} was found in 1978.
This Mersenne prime is far to big to be handled by normal every day computers and standard programming tools.

The base 10^8 positional numeral system can be used to check whether M_{21701} is indeed a prime number.
In the base 10^8 positional numeral system the Mersenne prime M_{21701} is depicted as:

$M_{21701} = g = G[\ 00{,}044{,}867 : 91{,}661{,}190 : 43{,}334{,}794 : \ldots : 09{,}285{,}741 : 08{,}283{,}535 : 11{,}882{,}751\ ]$ (base 10^8)
with $G(816) =$ "00,044,867" $G(815) =$ "91,661,190" $G(814) =$ "43,334,794"
. . .
$G(002) =$ "09,285,741" $G(001) =$ "08,283,535" $G(000) =$ "11,882,751"

$(10^8)^{816}$		$(10^8)^{815}$		$(10^8)^{814}$			$(10^8)^2$		$(10^8)^1$		$(10^8)^0$
G(816) 00044867	+	G(815) 91661190	+	G(814) 43334794	+ ... +		G(002) 09285741	+	G(001) 08283535	+	G(000) 11882751

Fig. 3.1b: The 25th Mersenne prime written in the base 10^8 positional numeral system.

The natural number M_{21701} is partitioned right-to-left into 817 positions, each with a value a_n with $0 \leq a_n < 10^8$.
For $M_{21701} = 44{,}867 \bullet 10^{6528}$ the number of positions needed is 6,528 / 8 + 1 = 816 + 1 = 817.
In the base 10^8 positional numeral system M_{21701} no longer is an integer.

3.2. Checking large integers against divisors.

3.2.1. Checking a natural number against $d = 2$ and $d = 5$.

In the base 10^8 positional numeral system, the easiest tests for primality are $d = 2$ and $d = 5$.
When testing $g = M_{21701}$ for primality it is enough to verify $G(000) \pmod 2 \neq 0$ and $G(000) \pmod 5 \neq 0$.
So, only the least significant digit is used, e.g. $G_0 = 11{,}882{,}751 \equiv 1 \pmod 2$ and $G_0 = 11{,}882{,}751 \equiv 1 \pmod 5$.
M_{21701} has no divisors $d = 2$ or $d = 5$, as expected.

3.2.2. Checking a natural number against the divisor $d = 3$.

The chapter "5.1. Using the Sieve of Eratosthenes to mark multiples of a divisor" describes how to test a large natural number in the base 10^2 positional numeral system against a divisor like $d = 7$.

Testing $g = M_{21701} = 44{,}867 \bullet 10^{6528}$ for $d = 3$ in a base 10^8 positional numeral system, first requires the value $g_c = [00{,}044{,}867]$ of the leading position.
Define $d = 3$ and $\Delta = 2d$ with Δ the distance between consecutive odd multiples of d.
Use $D = d + k \bullet \Delta$ to go up to g_c, via continuously adding Δ's.
The proces stops when $d + 7{,}477 \bullet \Delta < g_N < d + 7{,}478 \bullet \Delta$. So when $[00{,}044{,}865] < g_c < [00{,}044{,}871]$ (base 10^8).

Take the proces up one position, and add the divisor.
The next value of D becomes $[00{,}044{,}865 : 00{,}000{,}003]$ to check against $g_c = [00{,}044{,}867 : 91{,}661{,}190]$ (base 10^8).
Continue this proces until $D_n < M_{21701} \leq D_{n+1}$.

3.2.3. Checking a natural number against other divisors.

Given the natural number $g = M_{21701} = 44{,}867 \bullet 10^{6528}$ (the 25th Mersenne prime) with
$g = M_{21701} = G[\ 00{,}044{,}867 : 91{,}661{,}190 : 43{,}334{,}794 : 95{,}141{,}035 : \ldots : 08{,}283{,}535 : 11{,}882{,}751\ ]$ (base 10^8)

To check M_{21701} for primality, a list of all prime numbers up to $\sqrt{g}$ is needed.
Approximate the square root in a base $10\text{^}m$ positional numeral system with m is even, by using leading positions.
The last divisor d is just shy of $d = \sqrt{g} < \sqrt{44{,}868} \cdot \sqrt{10^{6{,}528}} < 211.83 \cdot 10^{3{,}264}$.

The $P_9\#$–Sieve only contains a list of prime numbers up to $p_9\# = p_1 \bullet p_2 \bullet \ldots \bullet p_9 = 223{,}092{,}870$, with
$\pi(p_9\#) = 12{,}283{,}531$ and $\pi(x)$ the prime count function.
The $P_9\#$–Sieve is the last primorial sieve to store its prime numbers in long words when using computers.

The extended $P_9\#$–Sieve holds the list of $\pi(10^9) = 50{,}847{,}534$ prime numbers up to 10^9.
Larger prime numbers, with $p_n > 10^9$, are evenly distributed above the $\varphi(p_9\#) = 36{,}495{,}360$ struts of the $P_9\#$–Sieve, see also the $P_3\#$–Sieve with its $\varphi(p_3\#) = \varphi(30) = 8$ struts in fig. A.1a.

When $d > 10^9$ the next possible divisor is found using the distance between the two corresponding struts $S(p_9\#)_{n-1}$ and $S(p_9\#)_n$ from the $P_9\#$–Sieve. Note that for large numbers most divisors will be composite numbers.
The next possible d_k for large d's is equal to $d_k = d_{k-1} + S_n - S_{n-1}$ with $d_k \equiv S(p_9\#)_n \pmod{p_9\#}$.
and $d_k = S_n + m \bullet p_9\#$.

While moving along all possible divisors $10^9 < d \leq \sqrt{g}$ above the $\varphi(p_9\#)$ struts, one of the divisors is
$M_{89} = 2^p - 1$, with $p = 89$ a prime number. M_{89} is the 10th Mersenne prime (1911), with $M_{89} < \sqrt{g}$ (see above).

Checking M_{21701} against the divisor $M_{89} = D[\ 00{,}000{,}618 : 97{,}001{,}964 : 49{,}562{,}111\ ]$ (base 10^8) first requires the value $g_c = [00{,}044{,}867]$, the leading position of M_{21701} when written in base 10^8.
Define $d = M_{89}$ and $\Delta = 2d$ the distance between consecutive odd multiples of d.
Since $d > g_c$ the positions of g_c must be moved three positions upwards, so select the leading four positions of g.
$g_c = G[\ 00{,}044{,}867 : 91{,}661{,}190 : 43{,}334{,}794 : 95{,}141{,}035\ ]$ (base 10^8) (see previous section)
Increasing D with Δ gives $D[\ 00{,}001{,}856 : 91{,}005{,}893 : 48{,}686{,}333\ ]$ (base 10^8) to be checked against g_c.
Keep increasing with Δ until $D_n < g_c \leq D_{n+1}$. Then move the positions of g_c one position upwards, etc.

4. Using the base 10^*m* numeral system.

4.1. The principles of the base 10^*m* numeral system.

A base 10^m numeral system is a positional number base system with a radix = 10^m.
This numeral system is similair to the well know Octal numeral system.
The base 10^m numeral system is only limited by the capabilities of the computer.

E.g. the natural number $g = 90{,}139_{10}$ = [09:01:39] (base 10^2), see figure 4.1a.
Note: $g = 90{,}139_{10}$ was checked against $d = 7$ in the section "the P_3#−sieve in a Cartesian coordinate system".

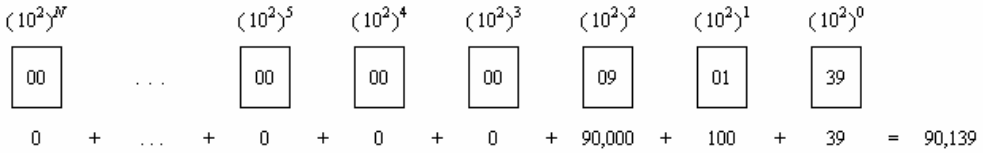

Fig. 4.1a: A base 10^2 numeral system, with number $g = 90{,}139_{10}$

4.2. Computational operations in a positional number base system.

4.2.1. Adding two numbers.

In every base numeral system adding two numbers is simple when remembering to carry the one.
For example in the normal base 10 right-to-left numeral system: $834{,}905{,}612_{10} + 65{,}127{,}834_{10} = 900{,}034{,}446_{10}$
In the base 10^2 numeral system, working from right-to-left, it becomes:

```
  1  1  1                    ← carry
[08:34:90:56:12] (base 10^2)
[00:65:12:78:34] (base 10^2)
--------------------------------- +
[09:00:03:34:46] (base 10^2)
```

Below is an example of adding $48{,}157_{10} + 791{,}802_{10} = 839{,}959_{10}$ in a right-to-left base 30 numeral system.

```
  1  1  1                    ← carry          1     1     ← carry
[00:01:23:15:07] (base 30)                =    48,157 10
[00:29:09:23:14] (base 30)                =   791,804 10
--------------------------------- +           ---------- +
[01:01:03:08:21] (base 30)                =   839,961 10
```

With g = [01:01:03:08:21] (base 30) = $1 \bullet (30)^4 + 1 \bullet (30)^3 + 3 \bullet (30)^2 + 8 \bullet (30)^1 + \bullet (30)^0 = 839{,}961_{10}$

4.2.2. Multiplying two numbers.

Multiplying two numbers in any base numeral system usually takes a lot of effort, but for exceptions like:
times 0, or times 1.

In the base 10^m numeral system multiplying with the base 10^m is identical with moving each position one upwards.
This specific feature significantly reduces steps when implementing the Sieve of Eratosthenes.

In the base 10^2 numeral system multiplying the number [00:56:12:78:34] (base 10^2) with 10^2 becomes:
[00:56:12:78:34] (base 10^2) • base 10^2 = [56:12:78:34:00] (base 10^2) = $5{,}612{,}783{,}400_{10}$
Each position is moved one position upwards, and the most right position gets the value 0_{10} = [00] (base 10^2).

4.3. Features of the 10^m numeral system.

4.3.1. Overview.

Many of the largest known primes are Mersenne primes, prime number's that are one less than a power of two and have the form $M_k = 2^k - 1$. If k is a composite number then so is the Mersenne prime.
The binary representation of M_k is composed of all 1's, since the binary form of $2^k - 1$ is simply k 1's.

In the year 2017 the largest known prime number was $M_{77232917} = 2^{77,232,917} - 1$, with 23,249,425 digits when written in base 10. This number, symply known as $M_{77232917}$, is the 50th rare Mersenne prime.

In the quest for ever bigger prime numbers a larger radix, like base 10^8, might be better suited then the base 10.
In the base 10^8 each position has an integer value a_n with $0 \leq a_n < b$ and $b = 10^8$.
Each value $a_n < 10^8$ easely fits in a double word with $(2^8)^4 = 2^{32}$ bits, since $10^8 = 10^2 \cdot (10^3)^2 \approx 2^7 \cdot (2^{10})^2 = 2^{27}$

The 50th rare Mersenne prime $M_{77232917} \approx 10^{24,000,000}$ with 24,000,000 digits
$M_{77232917}$ can be stored in a 4 byte integer array $D[0 : 10^9]$, with $D(0)$ the least significant position.
$D[0 : 10^9]$ will hold a number up to $(10^8)^{\wedge}10^9 = 10^{8,000,000,000}$ which is far greater then $M_{77232917} \approx 10^{24,000,000}$.

4.3.2. The square root of a natural number.

The Sieve of Eratosthenes stipulates that a natural number g must be checked up to the all divisors $< \sqrt{g}$.
E.g. for like $g = 2{,}345{,}607_{10} = [02:34:56:07]$ (base 10^2) the $\sqrt{g} = \sqrt{2{,}345{,}607} \approx 1{,}531_{10}$.

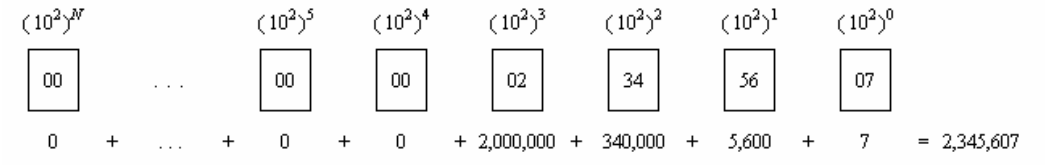

Fig. 4.3.2a: Example of a base 10^2 conversion to the decimal number $g = 2{,}345{,}607_{10}$

When dealing with the square root of large natural numbers, a 10^m numeral system with $m \in \{2, 4, 6, 8, ...\}$ should be used. Only a few leading positions are required to get a good estimate of the square root.
Thus in a base 10^2 numeral system $\sqrt{g} = \sqrt{2{,}345{,}607} \approx \sqrt{2.346} \cdot \sqrt{(10^2)^3} \approx 1.532 \cdot 10^3 = 1{,}532_{10}$.

In a base 10^8 numeral system the square root of $g = G[\, x : y : ... \,]$ with long word integers depends on the values in the leading positions.
For $G[\, x : y : ... \,]$ the leading positions are: $G(x_7 ; x_6 ; ... , x_1 ; x_0)_{(z)} + G(y_7 ; y_6 ; ... , y_1 ; y_0)_{(z-1)} + ...$
with (z) the leading or most significant position, and $(z - 1)$ the second most significant position
When approximating $\sqrt{g}$ with $G(x_7 ; x_6 ; ... , x_1 ; x_0)_{(z)} \geq G(00010000)$ use $G(x_7 ; x_6 ; ... , x_1 ; x_0 + 1)_{(z)}$,
otherwise use $G(x_3 ; x_2 ; x_1 ; x_0)_{(z)} \bullet 10^4 + G(y_7 ; y_6 ; y_5 ; y_4 + 1)_{(z-1)}$

5. The Sieve of Eratosthenes and the 10^*m* numeral system.

5.1. Using the Sieve of Eratosthenes to mark multiples of a divisor.

The simplist way to test if a natural number g is a prime number is trial division.
For every prime number between 2 and $\sqrt{g}$ check whether it is evenly divisible.

An alternative way to test a natural number is based on the Sieve of Eratosthenes.
For every prime number between 2 and $\sqrt{g}$ check if an odd multiple of a divisor is equal to the natural number g.
Per divisor d the single division operation is interchanged by multiple addition operations.
The main advantage beeing that the base 10^m numeral system can handle very large natural numbers.

The integer $g = 90{,}139_{10}$ was checked against $d = 7$ in the section "the P_3#−sieve in a Cartesian coordinate system".
When checking $g = 90{,}139_{10} = [09:01:39]$ (base 10^2) against the divisor $d = 7_{10} = [00:00:07]$ (base 10^2),
steps in the Sieve of Eratosthenes are to mark only odd multiples of $d = 7$, e.g. $d \in \{21, 35, 49, \ldots\}$.
The distance Δ between each odd multiple of $d = 7$ is $\Delta = 2 \bullet d = 14_{10} = [00:00:14]$ (base 10^2),
thus a translation over $\Delta = [00:00:14]$ (base 10^2).

The table below shows that for $d = 7$ it takes $(12{,}887 - 1) / 2 \approx 6{,}500$ addition steps to reach $g = 90{,}139_{10}$.

Begin (base 10^2)	Translation	End (base 10^2)	Comment
$D[00:00:07]_{(d,\ 1)}$	Δ	$D[00:00:21]_{(d,\ 3)}$	$(d, 3) = 7 \bullet 3 = 21$, third multiple of $d = 7$
$D[00:00:21]_{(d,\ 3)}$	Δ	$D[00:00:35]_{(d,\ 5)}$	$D[00:00:35]_{(7,\ 5)} = 35_{10}$ with $(d, 5)$ the fifth multiple
. . .			
$D[00:00:91]_{(d,\ 13)}$	Δ	$D[00:01:05]_{(d,\ 15)}$ [1]	$(d, 15) = 7 \bullet 15 = d + 7\Delta = 105_{10}$
. . .			
$D[00:08:89]_{(d,\ 127)}$	Δ	$D[00:09:03]_{(d,\ 129)}$ [1]	$D[00:09:03]_{(d,\ 129)} = 9 \bullet (10^2)^1 + 3 \bullet (10^2)^0 = 903$
. . .			
$D[08:99:15]_{(d,\ 12845)}$	Δ	$D[08:99:29]_{(d,\ 12847)}$ [2]	[2] $(d, 12847) = 7 \bullet 12{,}847 = d + 6{,}423\Delta = 89{,}929_{10}$
. . .			
$D[09:01:25]_{(d,\ 12875)}$	Δ	$D[09:01:39]_{(d,\ 12877)}$ [3]	[3] $d = 7 \bullet 12{,}877 = d + 6{,}438\Delta = 90{,}139_{10}$ For $d = 7$ then $d \mid 90{,}139_{10}$

[1] "Carry the one" in the right-to-left system, e.g. $D[00:00:105]_{(d,\ 15)} \equiv D[00:01:05]_{(d,\ 15)}$ (base 10^2) etc.
[2] For $d = 7$ the number $g_{10} = 89{,}929$ is the first number with $d \mid g_{10}$ in the interval [89,880 ; 90,149], see fig. A.1a.
[3] For $d = 7$ the number $g_{10} = 90{,}139$ is the second / last number with $d \mid g_{10}$ in the interval [89,880 ; 90,149], see fig. A.1a.

Note:
The original Sieve of Eratosthenes will not suffice when dealing with very large integers, since it requires an enormous amount of memory to set up the initial list of all integers up to $\sqrt{g}$.
One of the options is to use the ninth primorial sieve, the P_9# −Sieve, to approximate divisors.

5.2. Advancements to the Sieve of Eratosthenes principles.

5.2.1. Accelerating marking multiples of divisors.

In this study a large integer g is checked for primality using the principles of the Sieve of Eratosthenes.
For every divisor $d < \sqrt{g}$ odd multiples of the divisor, up to the natural number g, are compared with g.
Per divisor d the single division operation g / d is interchanged by multiple addition operations.
E.g. check if $d + n \bullet \Delta = g$, with $\Delta = 2 \bullet d$ for the odd multiples of d, and $n = \{0, 1, 2, 3, ...\}$
Ideally the divisors are only prime numbers.
The divisors can be approximated using the struts of the primorial sieve P_9#−Sieve.

The base 10^m numeral system is designed to store very large natural numbers into the memory of a computer.
This positional numeral system has unique features when using the principles of the Sieve of Eratosthenes.
In the base 10^8 right-to-left numeral system the integer g is broken up into eight digits per position.
Each odd multiple $D_{(d, n)}$ of a divisor d is checked against only leading positions of g.
As soon as the ever increasing $D_{(d, n)}$ is getting close, one more leading position of g is added.
The odd multiple $D_{(d, n)}$ is moved one position upwards to $D_{(d, 10^8 \bullet n)}$, and thus multiplied by the base 10^8.

E.g. given $g = 90{,}139_{10}$ in a base 10^2 numeral system the process as shown in the previous section is accelerated by first checking multipes of $d = 7$ against the most significant positions of $g = 90{,}139_{10}$ = [09:01:39] (base 10^2)

- Define $g_c = 9_{10}$ = [00:00:09] (base 10^2), the leading position of g (base 10^2)
- Select $D[00:00:07]_{(d, 1)}$ as the number just shy of $g_c = 9_{10}$ = [00:00:09] (base 10^2). See [1)] below.
  [2)] Move the position one upwards to $D[00:07:00]_{(d, 100)}$ and add $d = 7_{10}$ = [00:00:07] (base 10^2)
- [3)] The next starting point is $D[00:07:07]_{(d, 101)}$ to be checked against $g_c = 901_{10}$ = [00:09:01] (base 10^2)
- Etc, see table below.

The table below shows that for $d = 7$ it takes some 100 steps to reach $g = 90{,}139_{10}$ instead of the some 6,500 steps.

Begin (base 10^2)	Translation	End (base 10^2)	Comment
$D[00:00:07]_{(d, 1)}$ [1)]	Δ	$D[00:00:21]_{(d, 3)}$	$D[00:00:21]_{(d, 3)} > g_c$
$D[00:00:07]_{(d, 1)}$	times base	$D[00:07:00]_{(d, 100)}$ [2)]	move $D[00:00:07]_{(d, 1)}$ one base position upwards
$D[00:07:00]_{(d, 100)}$	add d	$D[00:07:07]_{(d, 101)}$ [3)]	New g_c = [00:09:01] of g = [09:01:39] (base 10^2)
$D[00:07:07]_{(d, 101)}$ [3)]	Δ	$D[00:07:21]_{(d, 103)}$	
. . .			► **Adding** 26 / 2 = 13 steps
$D[00:08:89]_{(d, 127)}$	Δ	$D[00:09:03]_{(7, 129)}$	$D[00:09:03]_{(7, 129)} > g_c$
$D[00:08:89]_{(d, 127)}$	times base	$D[08:89:00]_{(d, 12700)}$ [2)]	move $D[00:08:89]_{(d, 127)}$ one base position upwards ► **Leap** (100 − 1) • 127 / 2 steps
$D[08:89:00]_{(d, 12700)}$	add d	$D[08:89:07]_{(d, 12701)}$	New g_c = [09:01:39]
$D[08:89:07]_{(d, 12701)}$	Δ	$D[08:89:21]_{(d, 12703)}$	
. . .			► **Adding** 174 / 2 = 87 steps
$D[09:01:25]_{(7, 12875)}$	Δ	$D[09:01:39]_{(7, 12877)}$ [3)]	$D[09:01:39]_{(d, 12877)} = g_c$ $d = 7$ is indeed a divisor of g

5.2.2. The Sieve of Eratosthenes and segmentation.

When a significant range of large consecutive integers $[g_a ... g_b]$ is checked for primality, the basic features of the Sieve of Eratosthenes are ideally suited.
Compare every odd multiple of the divisor $d < \sqrt{g_a}$ against the natural number g_a.
Select the first integer in $[g_a ... g_b]$ that is evenly divisable by d and mark multiples of the divisor in the specified range.
At the end all unmarked integers in $[g_a ... g_b]$ then are prime numbers.

5.3. Verifying large natural numbers in the base 10^2 numeral system.

5.3.1. Checking a large integer against the divisor $d = 3$.

In this section the base 10^2 numeral system is used to illustrate the process of checking if a large natural number is a prime number. The natural number $g = 29{,}906{,}050{,}403 = [02:99:06:05:04:03]$ (base 10^2) is "randomly" choosen.
When using computers, an unsigned double word or long word has $(2^8)^4 = 2^{32}$ bits.
Since $2^{32} = 4{,}294{,}967{,}296 < g = 29{,}906{,}050{,}403$ long words are not enough for the calculations, hence the base 10^m.

It takes many steps for $d = 3$ to reach the first few checkpoints of $g = 29{,}906{,}050{,}403$ since g is just shy of $3 \bullet 10^{10}$.
The divisor $d = 3_{10} = [00:00:00:00:00:03]$ (base 10^2) is the first real divisor to be checked.

Steps in the Sieve of Eratosthenes are to mark only odd multiples of $d = 3$, e.g. $d \in \{d \bullet 3, d \bullet 5, d \bullet 7, \ldots\}$.
The distance Δ between each odd multiple of $d = 3$ is $\Delta = 2 \bullet d = 6_{10} = [00:00:00:00:00:06]$ (base 10^2),
thus a translation over $\Delta = [00:00:00:00:00:06]$ (base 10^2).
The lowest checkpoint is $g_c = [00:00:00:00:00:02]$, the leading position of $g = [02:99:06:05:04:03]$ (base 10^2)

Begin (base 10^2)	Translation	End (base 10^2)	Comment / (base 10^2)
$D[00:00:00:00:00:03]_{(d,\ 1)}$			$D[00:00:00:00:00:03]_{(d,\ 1)} > g_c$
			move g_c one position upwards New $g_c = [00:00:00:00:02:99]$
$D[00:00:00:00:00:03]_{(d,\ 1)}$	Δ	$D[00:00:00:00:00:09]_{(d,\ 3)}$	$(d, 3) = d \bullet 3 = 3 \bullet 3 = 9_{10}$
. . .			► **Adding** 98 / 2 = 49 steps
$D[00:00:00:00:02:97]_{(d,\ 99)}$	Δ	$D[00:00:00:00:03:03]_{(d,\ 101)}$	$D[00:00:00:00:03:03]_{(d,\ 101)} > g_c$
$D[00:00:00:00:02:97]_{(d,\ 99)}$	times base	$D[00:00:00:02:97:00]_{(d,\ 9900)}$	move one base position upwards ► **Leap** (100 – 1) • 99 / 2 steps
$D[00:00:00:02:97:00]_{(d,\ 9900)}$	add d	$D[00:00:00:02:97:03]_{(d,\ 9901)}$	New $g_c = [00:00:00:02:99:06]$
$D[00:00:00:02:97:03]_{(d,\ 9901)}$	Δ	$D[00:00:00:02:97:09]_{(d,\ 9903)}$	
. . .			► **Adding** 66 / 2 = 33 steps
$D[00:00:00:02:99:01]_{(d,\ 9967)}$	Δ	$D[00:00:00:02:99:07]_{(d,\ 9969)}$	$D[00:00:00:02:99:07]_{(d,\ 9969)} > g_c$
$D[00:00:00:02:99:01]_{(d,\ 9967)}$	times base	$D[00:00:02:99:01:00]_{(d,\ 996700)}$	move one base position upwards ► **Leap** (100 – 1) • 9,967 / 2 steps
$D[00:00:02:99:01:00]_{(d,\ 996700)}$	add d	$D[00:00:02:99:01:03]_{(d,\ 996701)}$	New $g_c = [00:00:02:99:06:05]$
$D[00:00:02:99:01:03]_{(d,\ 996701)}$	Δ	$D[00:00:02:99:01:09]_{(d,\ 996703)}$	
. . .			► **Adding** 166 / 2 = 83 steps
$D[00:00:02:99:06:01]_{(d,\ 996867)}$	Δ	$D[00:00:02:99:06:07]_{(d,\ 996869)}$	$D[00:00:02:99:06:07]_{(d,\ 996867)} > g_c$
$D[00:00:02:99:06:01]_{(d,\ 996867)}$	times base	$D[00:02:99:06:01:00]_{(d,\ 99686700)}$	move one base position upwards ► **Leap** 99 • 996,867 / 2 steps
$D[00:02:99:06:01:00]_{(d,\ 99686700)}$	add d	$D[00:02:99:06:01:03]_{(d,\ 99686701)}$	New $g_c = [00:02:99:06:05:04]$
$D[00:02:99:06:01:03]_{(d,\ 99686701)}$	Δ	$D[00:02:99:06:01:09]_{(d,\ 99686703)}$	
. . .			► **Adding** 132 / 2 = 66 steps
$D[00:02:99:06:04:99]_{(d,\ 99686833)}$	Δ	$D[00:02:99:06:05:05]_{(d,\ 99686835)}$	$D[00:02:99:06:05:05]_{(d,\ 99686835)} > g_c$
$D[00:02:99:06:04:99]_{(d,\ 99686833)}$	times base	$D[02:99:06:04:99:00]_{(d,\ 9968683300)}$	moved one base position upwards ► **Leap** 99 • 99,686,833 / 2 steps
$D[02:99:06:04:99:00]_{(d,\ 9968683300)}$	add d	$D[02:99:06:04:99:03]_{(d,\ 9968683301)}$	New $g_c = [02:99:06:05:04:03]$
$D[02:99:06:04:99:03]_{(d,\ 9968683301)}$	Δ	$D[02:99:06:04:99:09]_{(d,\ 9968683303)}$	
. . .			► **Adding** 166 / 2 = 53 steps
$D[02:99:06:05:04:01]_{(d,\ 9968683467)}$	Δ	$D[02:99:06:05:04:07]_{(d,\ 9968683469)}$	$D[02:99:06:05:04:07]_{(d,\ 9968683469)} > g_c$ $d = 3$ is NOT a divisor of g

To check the natural number $g = 29{,}906{,}050{,}403_{10}$ against the divisor $d = 3$ with the Sieve of Eratosthenes would need some 5,000 million steps. In the base 10^2 numeral system the number of operations is reduced to less then 1,000 instructions. More importantly, long words can be used to analyze the large integer $g = 29{,}906{,}050{,}403_{10}$.

5.3.2. Checking a large integer against a large divisor.

Assume that the "randomly" chosen g = 29,906,050,403 = [02:99:06:05:04:03] (base 10^2) has no divisors close to $\sqrt{g}$, then trial divisons must continue.
The last divisor for g is $d = \sqrt{g} < \sqrt{300 \cdot (10^2)^4} = 17.3205... \cdot 10^4$ = [00:00:00:17:32:05] (base 10^2).
Selecting the closest prime gives d = [00:00:00:17:28:01] (base 10^2) as a very large divisor to be checked.
This last check is only needed when still no divisor is found.

Check g against $d = 172{,}801_{10}$ = [00:00:00:17:28:01] (base 10^2). The smallest checkpoint g_c = [00:00:00:00:00:02]
Steps in the Sieve of Eratosthenes are to mark only odd multiples of d = 172,801, e.g. $d \in \{d \bullet 3, d \bullet 5, d \bullet 7, \ldots\}$.
The distance Δ between each odd multiple of d is $\Delta = 2 \bullet d = 345{,}602_{10}$ = [00:00:00:34:56:02] (base 10^2),
thus a translation over Δ = [00:00:00:34:56:02] .

Begin (base 10^2)	Translation	End (base 10^2)	Comment / (base 10^2)
D[00:00:00:17:28:01]$_{(d,\ 1)}$			D[00:00:00:17:28:01]$_{(d,\ 1)} > g_c$
			move position g_c 4 times one upwards New g_c = [00:00:02:99:06:05]
D[00:00:00:17:28:01]$_{(d,\ 1)}$	Δ	D[00:00:00:51:84:03]$_{(d,\ 3)}$	$d \bullet 3 = 172{,}801 \bullet 3 = 518{,}403_{10}$
D[00:00:00:51:84:03]$_{(d,\ 3)}$	Δ	D[00:00:00:86:40:05]$_{(d,\ 5)}$	
. . .			► **Adding** 14 / 2 = 7 steps
D[00:00:02:93:76:17]$_{(d,\ 17)}$	Δ	D[00:00:03:28:32:19]$_{(d,\ 19)}$	D[00:00:03:28:32:19]$_{(d,\ 19)} > g_c$
D[00:00:02:93:76:17]$_{(d,\ 17)}$	times base	D[00:02:93:76:17:00]$_{(d,\ 1700)}$	move one base position upwards ► **Leap** (100 – 1) • 17 / 2 steps
D[00:02:93:76:17:00]$_{(d,\ 1700)}$	add d	D[00:02:93:93:45:01]$_{(d,\ 1701)}$	New g_c = [00:02:99:06:05:04]
D[00:02:93:93:45:01]$_{(d,\ 1701)}$	Δ	D[00:02:94:28:01:03]$_{(d,\ 1703)}$	
. . .			► **Adding** 28 / 2 = 14 steps
D[00:02:98:77:29:29]$_{(d,\ 1729)}$	Δ	D[00:02:99:11:85:31]$_{(d,\ 1731)}$	D[00:02:99:11:85:31]$_{(d,\ 1731)} > g_c$
D[00:02:98:77:29:29]$_{(d,\ 1729)}$	times base	D[02:98:77:29:29:00]$_{(d,\ 172900)}$	move one base position upwards ► **Leap** (100 – 1) • 1,729 / 2 steps
D[02:98:77:29:29:00]$_{(d,\ 172900)}$	add d	D[02:98:77:46:57:01]$_{(d,\ 172901)}$	New g_c = [02:99:06:05:04:03]
D[02:98:77:46:57:01]$_{(d,\ 172901)}$	Δ	D[02:98:77:81:13:03]$_{(d,\ 172903)}$	
. . .			► **Adding** 164 / 2 = 82 steps
D[02:99:05:88:50:65]$_{(d,\ 173065)}$	Δ	D[02:99:06:15:06:67]$_{(d,\ 173067)}$	D[02:99:06:15:06:67]$_{(d,\ 173067)} > g_c$ d = 172,801 is NOT a divisor of g

The Sieve of Eratosthenes needs 173,067 / 2 steps to verify that d = 172,801 is not a divisor of g = 29,906,050,403.
The table above shows that less then 200 instructions are needed in the base 10^2 numeral position system.
More importantly, long words can be used to analyze the large integer g = 29,906,050,403.

6. Results of Prime numbers and base 10^*m* positional numeral systems.

This study "only" describes the genesis of a base 10^*m* positional numeral system. It is developed for verifying and finding ever larger prime numbers. The finds in this study are an offspring of the hunch to place a prime number on the short leg of a primitive Pythagorean triangle. No other records of a base 10^*m* positional numeral system where found, but for the base 100 positional numeral system by the name Centesimal.

This base 10^*m* positional numeral system accommodates mathematical operations on very large integers.
Using the ancient Sieve of Eratosthenes, it can check for large divisors that evenly devides an integer.
This document does not include computer simulations to determine its effectiveness.

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Appendix A: From the Primorial sieve towards a full numeral base system

A.1. Introduction

Studying the divisor $d = 7$ in the Primorial sieve $P_3\#-$sieve (see fig. A.1a and A.1b) gave an opening towards positional numeral base systems. The $P_3\#-$sieve was first placed in a Cartesian coordinate system, which evolved into a base $p_3\#$ numeral system, be it a left-to-right system. Switching to a right-to-left numeral base system paved the way to the base 10^m numeral systems.

A.1.1. Principles of the $P_3\#-$sieve.

The Primorial sieve $P_3\#-$sieve has a width of $p_3\# = 30$ and $\varphi(p_3\#) = 8$ struts, with $\varphi(m)$ Euler's totient function.
The primorial $p_3\#$ is the product of the first three prime numbers, e.g. $p_3\# = p_1 \bullet p_2 \bullet p_3 = 30$
The $\varphi(p_3\#) = 8$ struts $S(p_3\#)_j$ with $1 \le j \le \varphi(p_3\#)$ support the columns with possible prime numbers $> p_3$.
Prime numbers $> p_3\#$ can only be found above the $\varphi(p_3\#)$ struts, evenly distributed over the struts.
The $\varphi(p_3\#)$ struts are prime numbers, or multiples of prime numbers $> p_3$, while the strut $S(p_3\#)_1 = 1$.
In the $P_3\#-$sieve all struts $> p_3$ are prime numbers.

The $P_n\#-$sieve has $\varphi(p_n\#)$ struts of which $\pi(p_n\#) + n - 1$ are prime numbers, with $\pi(x)$ the prime-counting function.
The extended $P_9\#-$Sieve holds the list of $\pi(10^9) = 50{,}847{,}534$ prime numbers up to 10^9.

90121	90122	90123	90124	90125	90126	**90127**	90128	90129	90130	90131	90132	90133	90134	90135	90136	90137	90138	90139	90140	90141	90142	90143	90144	90145	90146	90147	90148	**90149**	90150
90091	90092	90093	90094	90095	90096	90097	90098	90099	90100	90101	90102	90103	90104	90105	90106	**90107**	90108	90109	90110	90111	90112	90113	90114	90115	90116	90117	90118	90119	90120
90061	90062	90063	90064	90065	90066	**90067**	90068	90069	90070	**90071**	90072	**90073**	90074	90075	90076	90077	90078	90079	90080	90081	90082	90083	90084	90085	90086	90087	90088	**90089**	90090
90031	90032	90033	90034	90035	90036	90037	90038	90039	90040	90041	90042	90043	90044	90045	90046	90047	90048	90049	90050	90051	90052	**90053**	90054	90055	90056	90057	90058	**90059**	90060
90001	90002	90003	90004	90005	90006	**90007**	90008	90009	90010	**90011**	90012	90013	90014	90015	90016	**90017**	90018	**90019**	90020	90021	90022	**90023**	90024	90025	90026	90027	90028	90029	90030
89971	89972	89973	89974	89975	89976	**89977**	89978	89979	89980	89981	89982	**89983**	89984	89985	89986	89987	89988	**89989**	89990	89991	89992	89993	89994	89995	89996	89997	89998	89999	90000
89941	89942	89943	89944	89945	89946	89947	89948	89949	89950	89951	89952	89953	89954	89955	89956	89957	89958	**89959**	89960	89961	89962	**89963**	89964	89965	89966	89967	89968	89969	89970
89911	89912	89913	89914	89915	89916	**89917**	89918	89919	89920	89921	89922	**89923**	89924	89925	89926	89927	89928	89929	89930	89931	89932	89933	89934	89935	89936	89937	89938	**89939**	89940
89881	89882	89883	89884	89885	89886	89887	89888	89889	89890	**89891**	89892	89893	89894	89895	89896	**89897**	89898	**89899**	89900	89901	89902	89903	89904	89905	89906	89907	89908	**89909**	89910
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
241	242	243	244	245	246	247	248	249	250	**251**	252	253	254	255	256	**257**	258	259	260	261	262	**263**	264	265	266	267	268	**269**	270
211	212	213	214	215	216	217	218	219	220	221	222	**223**	224	225	226	**227**	228	**229**	230	231	232	**233**	234	235	236	237	238	**239**	240
181	182	183	184	185	186	187	188	189	190	**191**	192	**193**	194	195	196	**197**	198	**199**	200	201	202	203	204	205	206	207	208	209	210
151	152	153	154	155	156	**157**	158	159	160	161	162	**163**	164	165	166	**167**	168	169	170	171	172	**173**	174	175	176	177	178	**179**	180
121	122	123	124	125	126	**127**	128	129	130	**131**	132	133	134	135	136	**137**	138	**139**	140	141	142	143	144	145	146	147	148	**149**	150
91	92	93	94	95	96	**97**	98	99	100	**101**	102	**103**	104	105	106	**107**	108	**109**	110	111	112	**113**	114	115	116	117	118	119	120
61	62	63	64	65	66	**67**	68	69	70	**71**	72	**73**	74	75	76	77	78	**79**	80	81	82	**83**	84	85	86	87	88	**89**	90
31	32	33	34	35	36	**37**	38	39	40	**41**	42	**43**	44	45	46	**47**	48	49	50	51	52	**53**	54	55	56	57	58	**59**	60
1	**2**	**3**	4	**5**	6	**7**	8	9	10	**11**	12	**13**	14	15	16	**17**	18	**19**	20	21	22	**23**	24	25	26	27	28	**29**	30
S_1						S_2				S_3		S_4				S_5		S_6				S_7						S_8	

Fig. A.1a: $P_3\#-$sieve: distribution of struts over the base $p_3\#$, with possible prime numbers $> p_3\#$ above the struts

For the $P_n\#-$sieve the principles of the Sieve of Eratosthenes state:
Check all natural numbers $> p_n\#$ up to the upperbound g against the divisors d with $p_n < d < \sqrt{g}$.
All divisors $p_n < p_k \le \sqrt{g}$ can be approximated with $d \in \{ S(p_n\#)_j + m \bullet p_n\# \mid 1 \le j \le \varphi(p_n\#) \wedge m \in \mathbf{N}_0 \}$.
There are no even multiples of d, since they are marked by the $P_1\#-$sieve, with $p_1 = 2$.

A.1.2. The P_3#−sieve in a Cartesian coordinate system.

The logical way to place the (double) Primorial sieve in a Cartesian coordinate system is to have the startvalue 1 at the coordinates (0, 0). All natural numbers g in the (double) Primorial sieve then have the coordinates $G(x_G, y_G)$ with $1 \le x_G \le 30$ and $y_G \ge 0$, with $g > 0$, see fig. A.1a.

Starting the P_3#−sieve with the value 0, the natural numbers g have the coordinates $G(x_G, y_G)$ with $0 \le x_G < 30$ and $y_G \ge 0$. The (double) P_3#−sieve thus contains all natural numbers ≥ 0.
E.g. the natural number $g = 89{,}929 \equiv 19 \pmod{p_3\#}$ above strut S_6 has the coordinates $G(x_G, y_G) = G(19, 2997)$.
This modified P_3#−sieve has the makings of a base p_3# numeral system, be it a left-to-right system.

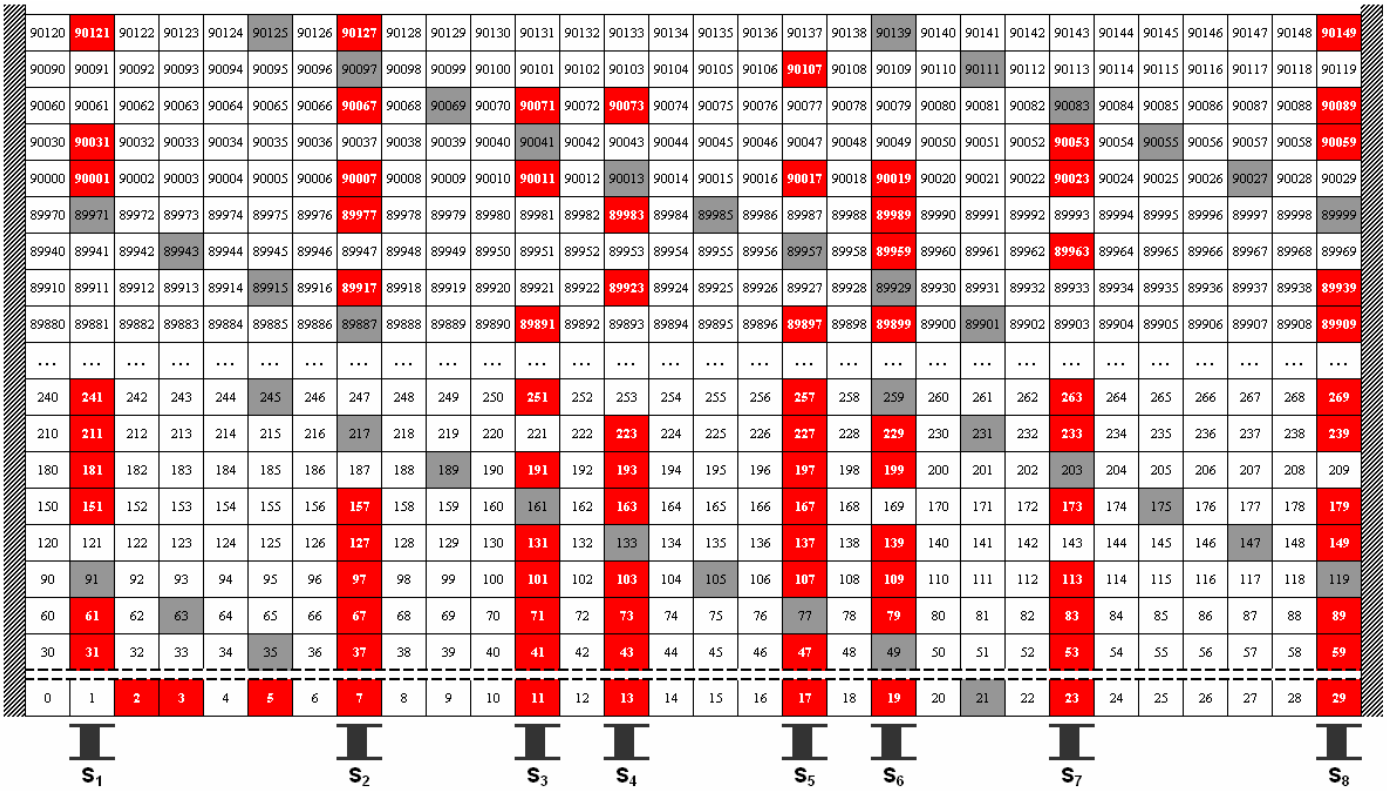

Fig. A.1b: P_3#−sieve: odd multiples of the divisor $d = 7$ in the base p_3# numeral system.

In the P_3#−sieve, the first divisor to be checked is $d = 7$, since $d > p_3$.
The divisor $d = 7$ is found at $D(x, y)_{(d, \#)} = D(x, y)_{(d, 1)} = D(7, 0)_{(d, 1)}$ with $d = x_{D(d, 1)} + y_{D(d, 1)} \bullet 30 = 7 + 0 \bullet 30 = 7$.
Steps in the Sieve of Eratosthenes are to mark multiples of d, e.g. $d \in \{14, 21, 38, 35, 42, 49, \ldots\}$.
The P_1#−sieve already eliminated even numbers $> p_1$, so only mark odd multiples of d, e.g. $d \in \{21, 35, 49, \ldots\}$.
The distance Δ between each odd multiple of $d = 7$ is $\Delta = 2 \bullet d = 14$, thus a translation over $D(x, y)_\Delta = D(14, 0)_\Delta$.

Begin ($d = 7$)	Translation	End	Comment
$D(7, 0)_{(d, 1)}$	$D(14, 0)_\Delta$	$D(21, 0)_{(d, 3)}$	$(d, 3) = 7 \bullet 3$, the third multiple of $d = 7$
$D(21, 0)_{(d, 3)}$	$D(14, 0)_\Delta$	$D(5, 1)_{(d, 5)}$ [1)]	$D(5, 1)_{(d, 5)} = x_D + y_D \bullet 30 = 5 + 1 \bullet 30 = 35 = d \bullet 5$
$D(5, 1)_{(d, 5)}$	$D(14, 0)_\Delta$	$D(19, 1)_{(d, 7)}$	$(d, 7) = 7 \bullet 7$, the 7th multiple of $d = 7$
$D(19, 1)_{(d, 7)}$	$D(14, 0)_\Delta$	$D(3, 2)_{(d, 9)}$ [1)]	$D(3, 2)_{(d, 9)} = x_D + y_D \bullet 30 = 3 + 2 \bullet 30 = 63 = d \bullet 9$
. . .			
$D(5, 2997)_{(d, 12845)}$ [2)]	$D(14, 0)_\Delta$	$D(19, 2997)_{(d, 12847)}$	$D(19, 2997)_{(d, 12847)} = 19 + 2{,}997 \bullet 30 = 89{,}929$
. . .			
$D(5, 3004)_{(7, 12875)}$ [2)]	$D(14, 0)_\Delta$	$D(19, 3004)_{(7, 12877)}$	$D(19, 3004)_{(7, 12877)} = 19 + 3{,}004 \bullet 30 = 90{,}139$

[1)] "Carry the one" in the left-to-right system, e.g. $D(35, 0)_{(d, 5)} \equiv D(5, 1)_{(d, 5)}$ (base p_3#) and $D(33, 1)_{(d, 9)} \equiv D(3, 2)_{(d, 9)}$ (base p_3#)
[2)] See the values $g = 89{,}929$ and $g = 90{,}139$ above S_6 in fig. A.1b

A.1.3. The P_3#−sieve: checking natural numbers for primality.

In the P_3#−sieve, the natural number $g = 90{,}109$ above strut S_6 (see fig. A.1b) has no divisors up to $d = 251$.
The divisor $d = 251 \bullet 1$ is found at $D(x, y)_{(d, \#)} = D(x, y)_{(251, 1)} = D(251, 0)_{(251, 1)} \equiv D(11, 8)_{(251, 1)}$ (base p_3#)
with $d = x_{D(251, 1)} + y_{D(251, 1)} \bullet 30 = 11 + 8 \bullet 30 = 251$, see also fig. A.1b.
Steps in the Sieve of Eratosthenes are to mark odd multiples of $d = 251$, e.g. $d \in \{251 \bullet 3, 251 \bullet 5, 251 \bullet 7, \ldots\}$.
The distance Δ between each odd multiple of $d = 251$ is $\Delta = 2 \bullet d = 502$.
$D(x, y)_\Delta = D(502, 0)_\Delta \equiv D(22, 16)_{(251, 2)}$ (base p_3#), thus a translation over $D(x, y)_\Delta = D(22, 16)_\Delta$.

Begin	Translation	End	Comment
$D(11, 8)_{(251, 1)}$	$D(22, 16)_\Delta$	$D(3, 25)_{(251, 3)}$ [1)]	$d = 251 \bullet 3$, the third multiple of $d = 251$
$D(3, 25)_{(251, 3)}$	$D(22, 16)_\Delta$	$D(25, 41)_{(251, 5)}$ [1)]	$D(25, 41)_{(251, 5)} = 25 + 41 \bullet 30 = 1{,}255$
. . .			
$D(27, 2986)_{(251, 357)}$	$D(22, 16)_\Delta$	$D(19, 3003)_{(251, 359)}$ [1)]	$D(19, 3003)_{(251, 359)} = 19 + 3{,}003 \bullet 30 = 90{,}109$ For $d = 251$ then $d \mid 90{,}109$

[1)] "Carry the one" in the left-to-right system,
e.g. $D(33, 24)_{(251, 3)} \equiv D(3, 25)_{(251, 3)}$ (base p_3#) and $D(49, 3002)_{(251, 359)} \equiv D(19, 3003)_{(251, 359)}$ (base p_3#)

Note: The table is based on an incomplete left-to-right base P_3# numeral system.

A.2. Switching towards a positional numeral base system.

A.2.1. The Primorial sieve as a partial base positional numeral system.

The p_3# positional numeral system, see chapter A.1., can be seen as a partial base positional numeral system.
It is limited to a two dimensional representation AND it is based on the Cartesian (x, y) coordinate system.

By interchanging the (x, y) coordinate system to a (y, x) coordinate system, the numeral system becomes an incomplete right-to-left base p_3# numeral system.

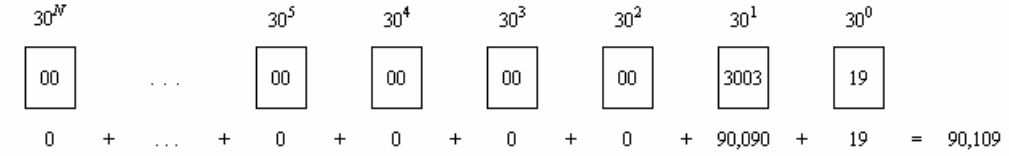

Fig. A.2a: Example of an incomplete right-to-left base p_3# numeral system.

In the incomplete right-to-left base p_3# numeral system, the number $g = 90{,}109_{10}$ is found at $D(3003, 19)_{(251, 359)}$
Steps in the Sieve of Eratosthenes are to mark odd multiples of $d = 251$, e.g. $d \in \{251 \bullet 3, 251 \bullet 5, 251 \bullet 7, \ldots\}$.
The distance Δ between each odd multiple of $d = 251$ is $\Delta = 2 \bullet d = 502$.
$D(y, x)_\Delta = D(0, 502)_\Delta \equiv D(16, 22)_{(251, 2)}$ (base p_3#), thus a translation over $D(y, x)_\Delta = D(16, 22)_\Delta$.

Begin	Translation	End	Comment
$D(8, 11)_{(251, 1)}$	$D(16, 22)_\Delta$	$D(25, 3)_{(251, 3)}$ [1)]	$d = 251 \bullet 3$, the third multiple of $d = 251$
$D(25, 3)_{(251, 3)}$	$D(16, 22)_\Delta$	$D(41, 25)_{(251, 5)}$	$D(41, 25)_{(251, 5)} = 41 \bullet 30^1 + 25 \bullet 30^0 = 1{,}255$
. . .			
$D(2986, 27)_{(251, 357)}$	$D(16, 22)_\Delta$	$D(3003, 19)_{(251, 359)}$ [1)]	$D(3003, 19)_{(251, 359)} = 3{,}003 \bullet 30^1 + 19 = 90{,}109_{10}$ For $d = 251$ then $d \mid 90{,}109$

[1)] "Carry the one" in the right-to-left system,
e.g. $D(24, 33)_{(251, 3)} \equiv D(25, 3)_{(251, 3)}$ (base p_3#) and $D(3002, 49)_{(251, 359)} \equiv D(3003, 19)_{(251, 359)}$ (base p_3#)

A.2.2. Implementation of a full base p_3# numeral system.

The incomplete right-to-left base p_3# numeral system can be made into a full right-to-left base p_3# numeral system. The natural number $g = 90{,}109$ becomes $g = 90{,}109_{10} = [03:10:03:19]$ (base p_3#), see below.

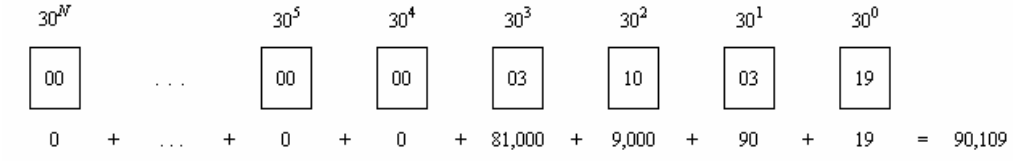

Fig. A.2b: Full implementation of a right-to-left base p_3# numeral system.

In the previous section $g = 90{,}109$ was checked against $d = 251$ using an incomplete base p_3# numeral system. In the full right-to-left base p_3# numeral system the divisor $d = 251_{10}$ is represented as [00:00:08:11] (base p_3#), with $d = 8 \bullet 30^1 + 11 \bullet 30^0 = 251$ and $g = 90{,}109_{10} = [03:10:03:19]$ (base p_3#).

Steps in the Sieve of Eratosthenes are to mark odd multiples of $d = 251$, e.g. $d \in \{251 \bullet 3, 251 \bullet 5, 251 \bullet 7, \ldots\}$. The distance Δ between each odd multiple of $d = 251$ is $\Delta = 502_{10} = [00:00:16:22]$ (base p_3#).

Begin	Translation	End	Comment
$D[00:00:08:11]_{(251,\ 1)}$	Δ	$D[00:00:25:03]_{(251,\ 3)}$ [1)]	$d = 251 \bullet 3 = 753$, the 3d multiple of $d = 251$
$D[00:00:25:03]_{(251,\ 3)}$	Δ	$D[00:01:11:25]_{(251,\ 5)}$ [1)]	$1 \bullet 30^2 + 11 \bullet 30^1 + 25 \bullet 30^0 = 1{,}255$
. . .			
$D[03:09:16:27]_{(251,\ 357)}$	Δ	$D[03:10:03:19]_{(251,\ 359)}$	$D[3:10:3:19]_{(251,\ 359)} = 90{,}109_{10}$ For $d = 251$ then $d \mid 90{,}109$

[1)] "Carry the one" in the right-to-left system, e.g. $D[00:00:24:33]_{(251,\ 3)} \equiv D[00:00:25:03]_{(251,\ 3)}$ (base p_3#) etc.

A.3. Evolving to a base 10^*m* positional numeral system.

Once the full implementation of a right-to-left base p_3# numeral system was in place, evolving to a base 10^*m* positional numeral system was a natural step.

In the base 10^*m* positional numeral system the ancient Sieve of Eratosthenes turns out to be very powerfull.