

## Prime number projects for students.

The (double) Primorial sieve and the unraveled Ulam spiral can offer a wide range of challenging projects for students how study math or computer science.

Below are some possible studies.

### Implementing the eight families of functions to build the Ulam spiral.

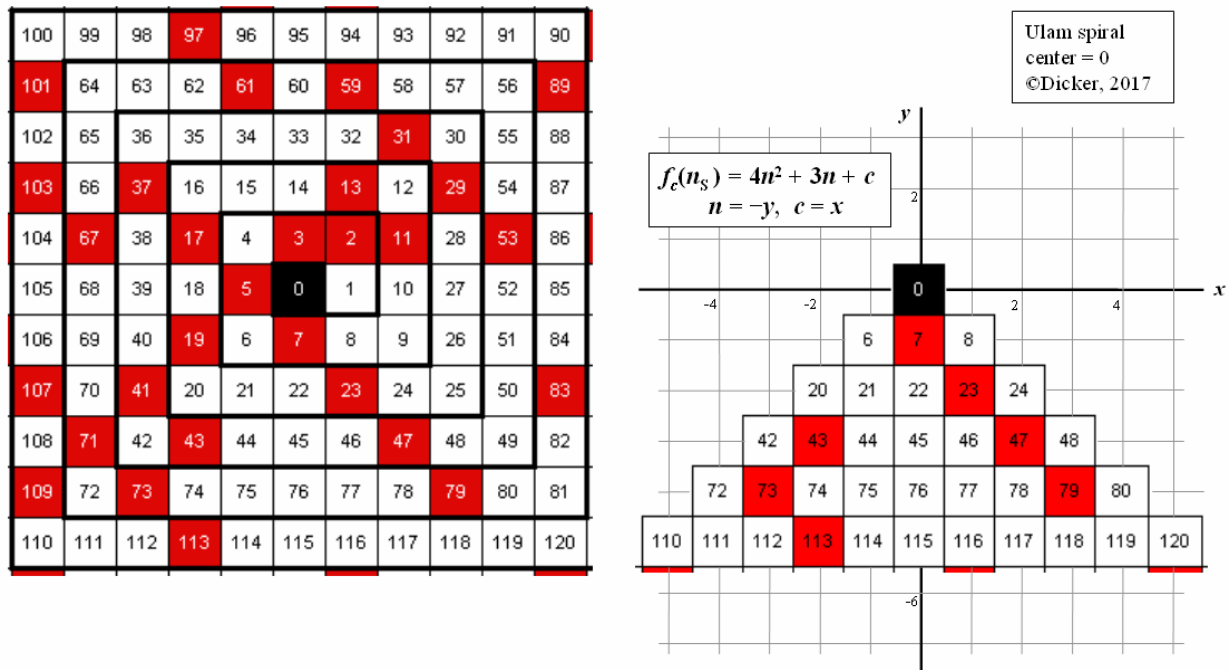


Fig. 1: Implementing four of the eight families of functions.

The traditional way to build a counterclockwise Ulam spiral with startvalue 0 is via the spiral.

The Sieve of Eratosthenes supplies a list of all prime numbers up to the last natural number in the spiral.

When placing the Ulam spiral with startvalue 0 in the Cartesian coordinate system, the spiral is fully defined by just eight families of functions. This offers the possibility to build the complete spiral via the E, N, S and W sectors. The Primorial sieve can be used to generate the list of all prime numbers up to the last natural number in the spiral.

Fig. 1 shows how to implement the S sector.

Project:

Build a 400 x 400 counterclockwise Ulam spiral with startvalue 0 via both methods.

The 41th SW diagonal appears to be rich with prime numbers.

Use both spirals to find all prime numbers < 400 · 400 on just this diagonal.

Compare both methods.



### Prime number density on the 59th SE diagonal.

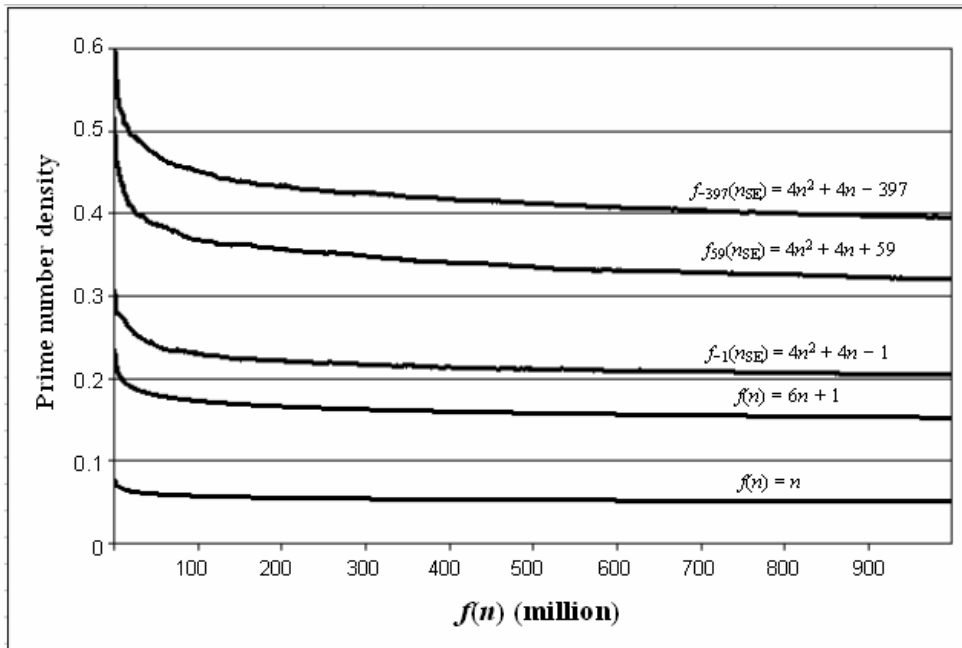


Fig. 3: Prime numbers density in different functions.

The study into the unraveling of the Ulam spiral presents the conjecture that diagonals with prime numbers  $> p_4$  contain infinitely many prime numbers, which answers one of the questions of Ulam and his team.

Fig. 3 shows for different functions the ratio  $r(f(n))$  of prime numbers in  $f(n)$  up to  $f(n) = 10^9$ .

The  $r(f_{b,c}(n) = 4n^2 + bn + c)$  with  $-3 \leq b \leq 4$  approaches  $C_{b,c} \cdot r(f(n) = n)$  by equal function values, with  $C_{b,c}$  a constant in  $\mathbf{R}_{\geq 0}$ .

Project:

Verify for the 59th SE diagonal up to  $10^{10}$  if  $C_{b,c} = C_{4,59} = 6.3$ , with  $f_{59}(n_{SE}) = 4n^2 + 4n + 59$

Build a computer program and use the 9th (double) Primorial sieve when checking for prime numbers.

Use 4 Byte integers for the value of the struts of the 9th Primorial sieve.

## First prime number $> 10^{16}$ in each sector.

For any natural number  $g$  in the Ulam 0–spiral the coordinates in the Cartesian coordinate system can be calculated through the families of functions. Define  $m = \sqrt{g/4}$  with  $m \in \mathbf{R}_{\geq 0}$  and  $n = \lfloor m \rfloor$  with  $n \in \mathbf{N}_0$ .

The value  $m - n$  determines the sector in which  $g$  lies, see the table below.

$m - n$	Sector	Function	( $x$ , $y$ )
$-1/2 \leq m - n < -1/4$	E	$f_c(n_E) = 4n^2 - 3n + c$	( $n$ , $c$ )
$-1/4 \leq m - n < 0$	N	$f_c(n_N) = 4n^2 - 1n + c$	( $-c$ , $n$ )
$0 \leq m - n < 1/4$	W	$f_c(n_W) = 4n^2 + 1n + c$	( $-n$ , $-c$ )
$1/4 \leq m - n < 1/2$	S	$f_c(n_S) = 4n^2 + 3n + c$	( $c$ , $-n$ )

The natural number  $g = 1.0 \cdot 10^{16}$  has the coordinates  $(x, y) = (-5.0 \cdot 10^7, 5.0 \cdot 10^7)$  and lies in the W sector.

The first prime number  $> 1.0 \cdot 10^{16}$  is probably found in the W sector with the coordinates

$(x, y) = (-5.0 \cdot 10^7, 5.0 \cdot 10^7 - \delta)$ , with  $\delta \in \mathbf{Z}$  and  $-5.0 \cdot 10^7 < \delta < 5.0 \cdot 10^7$ . Most likely applies  $0 < \delta < 1\% \cdot 5.0 \cdot 10^7$ .

Project:

Build a computer program to find the first prime number  $> 1.0 \cdot 10^{16}$  in each E, N, W and S sector (see above).

Use the 9th (double) Primorial sieve with 4 Byte integers when checking for prime numbers.

Prime numbers just over  $10^{16}$  are selected since 8 Byte integers should suffice, and because  $(p_{9\#})^2 = 5.0 \cdot 10^{16}$ .

Can the same prime numbers be found using the traditional Ulam spiral?

## Large sequences of prime numbers up to $10^9$ .

Large sequences of prime numbers are only found on diagonals. Each diagonal begins at the last intersection with the line  $|y| = |x|$ . The SW diagonal of the function  $f_{41}(n_{SW}) = 4n^2 + 2n + 41$  starts at  $n_{SW} = 21$ , see the figure below. The natural number 1681 is positioned in the E sector, and is thus no member of the SW diagonal.

The function  $f_{41}(n_{SW}) = 4n^2 + 2n + 41$  has 10 consecutive prime numbers in  $71 \leq n_{SW} \leq 80$  with the sequence  $\{20347, \dots, 25801\}$

The function  $f_{59}(n_{SE}) = 4n^2 + 4n + 59$  has 13 consecutive prime numbers in  $366 \leq n_{SE} \leq 378$  with the sequence  $\{537347, \dots, 573107\}$

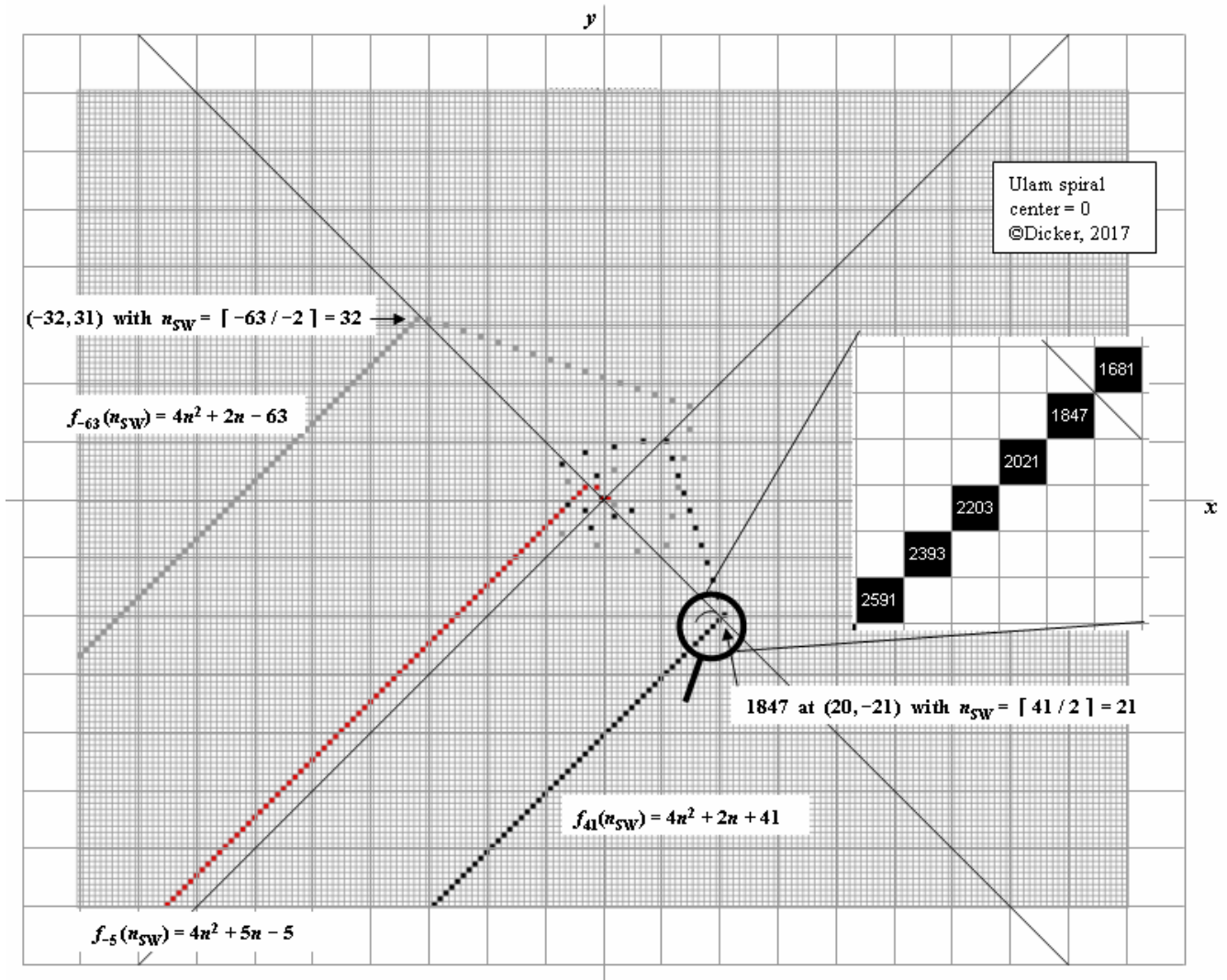


Fig. 4: Some SW diagonals and there corresponding members of the eight families of functions.

Project:

Design a method to find large sequences of prime numbers  $< 10^9$  via the traditional Ulam spiral. Check to see if all members in the sequence belong to the same diagonal.

Use the eight families of functions to find large sequences of prime numbers  $< 10^9$  and verify the results.

## **Results, questions and suggestions.**

Results, questions and suggestions can be sent to: [info@primorial-sieve.com](mailto:info@primorial-sieve.com)

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